

# Accuplacer College Level Math Study Guide

The following sample questions are similar to the format and content of questions on the Accuplacer College Level Math test. Reviewing these samples will give you a good idea of how the test works and just what mathematical topics you may wish to review before taking the test itself. Our purposes in providing you with this information are to aid your memory and to help you do your.

## Section One: Factoring and expanding polynomials.

Factor the following polynomials:

### Question One:

Fifteen A to the third power B to the second power minus forty five A to the second power B to the third power minus sixty A to the second power B.

### Answer One:

The answer is fifteen A to the second power times B times (AB minus three B to the second power minus four).

### Steps to the answer:

When factoring, there are three steps to keep in mind.

One. Always factor out the Greatest Common Factor.

Two. Factor what is left.

Three. If there are four terms, consider factoring by grouping.

### Question Two:

Seven X to the third power Y to the third power plus twenty one X to the second power Y to the second power minus ten X to the third power Y to the second power minus thirty X to the second power Y.

### Answer Two:

The answer is X to the second power times Y times (seven Y minus ten) times (X Y plus three).

### Steps to the answer:

Since there are four terms, we consider factoring by grouping.

First, take out the Greatest Common Factor, which is X to the second power times Y.

This results in the expression of X to the second power times Y times (seven X times Y to the second power plus twenty one Y minus ten X Y minus thirty).

When you factor by grouping, be careful of the minus sign between the two middle terms.

Using brackets and parentheses we can group the problem so it becomes X to the second power times Y times [(seven X times Y to the second power plus twenty one Y) plus (negative ten X Y minus thirty)].

Factoring further results in the expression of X to the second power times Y [seven Y times (X Y plus three) minus ten times (X Y plus three)].

Factoring the binomial (X Y plus three) results in the answer of X to the second power times Y times (seven Y minus ten) times (X Y plus three).

### Question Three:

Six X to the fourth power Y to the fourth power minus six X to the third power Y to the second power plus eight X Y to the second power minus eight.

### Answer Three:

The answer is two times three X to the third power times Y to the second power plus four) times (X times Y to the second power minus one).

### Question Four:

Two X to the second power minus seven X Y plus six Y to the second power.

### Answer Four:

The answer is (two X minus three Y) times (X minus two Y).

**Question Five:**

$Y$  to the fourth power plus  $Y$  to the second power minus six.

**Answer Five:**

The answer is ( $Y$  to the second power minus two) times ( $Y$  to the second power plus three).

**Steps to the answer:**

When a problem looks slightly odd, we can make it appear more natural to us by using substitution (a procedure needed for calculus). Let  $U$  equal  $Y$  to the second power. Substitute  $U$  in the original problem which results in the expression becoming  $U$  to the second power plus  $U$  minus six.

Factor the expression with the letter  $U$ 's. This results in ( $U$  minus two) times ( $U$  plus three).

Then, substitute the  $Y$  to the second power back in place of the letter  $U$ 's. This results in the answer of ( $Y$  to the second power minus two) times ( $Y$  to the second power plus three).

If you can factor more, proceed. Otherwise you are done.

**Question Six:**

Seven  $X$  to the third power plus fifty six  $Y$  to the third power.

**Answer Six:**

The answer is seven ( $X$  plus two  $Y$ ) times ( $X$  to the second power minus two  $X Y$  plus four  $Y$  to the second power).

**Steps to the answer:**

Use the formula for factoring the sum of two cubes:

$A$  to the third power plus  $B$  to the third power equals ( $A$  plus  $B$ ) times ( $A$  to the second power minus  $A B$  plus  $B$  to the second power).

The formula for the difference of two cubes is:

$A$  to the third power minus  $B$  to the third power equals ( $A$  minus  $B$ ) times ( $A$  to the second power plus  $A B$  plus  $B$  to the second power).

**Question Seven:**

Eighty one  $R$  to the fourth power minus sixteen  $S$  to the fourth power.

**Answer Seven:**

The answer is (three  $R$  minus two  $S$ ) times (three  $R$  plus two  $S$ ) times (nine  $R$  to the second power plus four  $S$  to the second power).

**Question Eight:**

$(X$  plus  $Y)$  to the second power plus two times  $(X$  plus  $Y)$  plus one.

**Answer Eight:**

The answer is  $(X$  plus  $Y$  plus one) to the second power.

**Hint:**

Let  $U$  equal  $X$  plus  $Y$ .

Expand the following:

**Question Nine:**

$(X$  plus one) times  $(X$  minus one) times  $(X$  minus three).

**Answer Nine:**

The answer is  $X$  to the third power minus three  $X$  to the second power minus  $X$  plus three.

**Question Ten:**

(two  $X$  plus three  $Y$ ) to the second power.

**Answer Ten:**

The answer is for  $X$  to the second power plus twelve  $X Y$  plus nine  $Y$  to the second power.

**Question Eleven:**

$(X$  times square root of three plus square root of three) times ( $x$  times square root of six minus the square root of six).

**Answer Eleven:**

The answer is three times the square root of two times  $X$  to the second power minus three times the square root of two.

**Question Twelve:**

$(X$  to the second power minus two  $X$  plus three) to the second power.

**Answer Twelve:**

The answer is  $X$  to the fourth power minus four  $X$  to the third power plus ten  $X$  to the second power minus twelve  $X$  plus nine.

**Question Thirteen:**

( $X$  plus one) to the fifth power.

**Answer Thirteen:**

The answer is  $X$  to the fifth power plus five  $X$  to the fourth power plus ten  $X$  to the third power plus ten  $X$  to the second power plus five  $X$  plus one.

**Steps to the answer:**

When doing problems thirteen and fourteen, you may want to use Pascal's triangle.

The triangle is formed using numbers.

The first row contains the number one.

The second row contains the number one followed by the number one.

The third row contains the number one followed by the number two followed by the number one.

The fourth row contains the number one followed by the number three followed by the number three followed by the number one.

The fifth row contains the number one followed by the number four followed by the number six followed by the number four followed by the number one.

Only the first five rows are given in the study guide.

**Question Fourteen:**

( $X$  minus one) to the six power.

**Answer Fourteen:**

The answer is  $X$  to the six power minus six  $X$  to the fifth power plus fifteen  $X$  to the fourth power minus twenty  $X$  to the third power plus fifteen  $X$  of the second power minus six  $X$  plus one.

## Section Two: Simplification of Rational Algebraic Expressions.

Simplify the following. Assume all variables are larger than zero.

**Question One:**

Three to the second power plus five minus square root of four plus four to the zero power.

**Answer One:**

The answer is thirteen.

**Question Two:**

Nine divided by three times five minus eight divided by two plus twenty seven.

**Answer Two:**

The answer is thirty eight.

**Question Three:**

The square root of the fraction eighty one over  $X$  to the fourth power.

**Answer Three:**

The answer is the fraction nine over  $X$  to the second power.

**Hint:**

If you have the square root of four you can write for as a product of primes which would be (two times two). In square roots, it takes two of the same thing on the inside to get one thing on the outside: such as the square root of four equals the square root of (two times two) equals two.

**Question Four:**

Two times the square root of eighteen minus five times the square root of thirty two plus seven times the square root of one hundred sixty two.

**Answer Four:**

The answer is forty nine times the square root of two.

**Question Five:**

The following problem contains two fractions separated by multiplication.

The first fraction is six  $X$  minus eighteen all over three  $X$  the second power plus two  $X$  minus eight.

The second fraction is twelve  $X$  minus sixteen all over for  $X$  minus twelve.

**Answer Five:**

The answer is the fraction six over ( $X$  plus two).

## Section Three: Solving Equations.

### Part A. Solving Linear Equations.

**Question One:**

Three minus two times (X minus one) equals X minus ten.

**Answer One:**

The answer is X equals five.

**Question Two:**

X over two minus X over seven equals one.

**Answer Two:**

The answer is X equals the fraction fourteen over five which is equivalent to the mixed number of two and four fifths.

**Question Three:**

Y times (Y plus two) equals Y to the second power minus six.

**Answer Three:**

The answer is Y equals negative three.

**Question Four:**

Two times [X minus (one minus three X)] equals three times (X plus one).

**Answer Four:**

The answer is X equals one.

## Part B. Solving Quadratic and Polynomial Equations.

Steps to solving quadratic score polynomials:

- One. Try to factor.
- Two. If factoring is not possible, use the quadratic formula, which is the fraction:  $X$  equals negative  $B$  plus or minus the square root of ( $B$  to the second power minus four  $A C$ ) all over two times  $A$ , where the quadratic equation is written in the form  $AX$  to the second power plus  $BX$  plus  $C$  equals zero.

Also note:  $i$  equals the square root of negative one. For example: the square root of negative twelve equals  $i$  times the square root of twelve equals  $i$  times the square root of (two times two times three) equals two times  $i$  times the square root of three.

### Question One:

$(Y - \frac{8}{3}) \times (Y + \frac{2}{3}) = 0$ .

### Answer One:

The answer is  $Y = \frac{8}{3}$  and  $-\frac{2}{3}$ .

### Question Two:

$2X^3 - 4X^2 - 30X = 0$ .

### Answer Two:

The answer is  $X = 0, -3, 5$ .

### Question Three:

$27X^3 = 1$ .

### Answer Three:

The answer is  $X = \frac{1}{3}$  and  $(-\frac{1}{6} \pm i\sqrt{\frac{3}{4}})$ .

### Question Four:

$(X - 3)(X + 6) = 9X + 22$ .

### Answer Four:

The answer is  $X = 10$  and  $-4$ .

### Question Five:

$T^2 + T = 0$ .

### Answer Five:

The answer is  $T = -\frac{1}{2} \pm i\sqrt{\frac{3}{4}}$ .

### Question Six:

$3X^3 = 24$ .

### Answer Six:

The answer is  $X = 2 \pm i\sqrt{3}$ .

### Question Seven:

$(X + 1)^2 + X^2 = 25$ .

### Answer Seven:

The answer is  $X = 3$  and  $-4$ .

### Question Eight:

$5Y^2 - Y = 1$ .

### Answer Eight:

The answer is  $Y = \frac{1 \pm \sqrt{21}}{10}$ .

## Part C. Solving Rational Equations.

Steps to solving rational equations:

- One. Find the lowest common denominator for all fractions in the equation.
- Two. Multiply both sides of the equation by the lowest common denominator.
- Three. Simplify and solve for the given variable.
- Four. Check answers to make sure that they do not cause the zero to occur in the denominators of the original equation.

### Question One:

One over (Y minus one) plus two over (Y plus one) equals zero.

### Answer One:

The answer is Y equals one over three.

### Steps to the answer:

First, multiply both sides of the equation by the lowest common denominator, which is (Y minus one) times (Y plus one).

This results in the equation becoming (Y minus one) times (Y plus one) times [one over (Y minus one) plus two over (Y plus one)] equals zero times (Y minus one) times (Y plus one).

Using the distributive property, the equation becomes (Y minus one) times (Y plus one) times one over (Y minus one) plus (Y minus one) times (Y plus one) times two over (Y plus one) equals zero.

Simplifying the fractions results in the equation of (Y plus one) plus two times (Y minus one) equals zero.

Using the distributive property and combining like terms results in the equation of three Y minus one equals zero.

Solve this equation to get the answer of the Y equals one over three.

### Question Two:

Two over (X minus three) minus three over (X plus three) equals twelve over (X to the second power minus nine).

### Answer Two:

The answer is no solution.

### Hint:

Working the problem, we get X equals three. However, three causes the denominators to be zero in the original equation. Hence, this problem has no solution.

### Question Three:

One over (six minus X) plus two over (X plus three) equals five X over (X of the second power minus three X minus eighteen).

### Answer Three:

The answer is X equals negative fifteen over four.

### Question Four:

Eleven over (X to the second power minus twenty five) minus two over (X minus five) equals one over (X plus five).

### Answer Four:

The answer is X equals two.

### Question Five:

One over A equals negative six over (A to the second power plus five).

### Answer Five:

The answer is A equals negative one and negative five.

### Question Six:

Negative one over (X to the second power minus three X) equals one over X plus X over (X minus three).

### Answer Six:

The answer is X equals negative two and one.

## Part D. Solving Absolute Value Equations.

Steps to solving absolute value equations:

- One. Isolate the absolute value on one side of the equation and everything else on the other side.
- Two. Remember that the absolute value of  $X$  equals two means that the object inside the absolute value has a distance of two away from zero. The only numbers with a distance of two away from zero are two and negative two. Hence,  $X$  equals two or  $X$  equals negative two. Use the same thought process for solving other absolute value equations.

### Question One:

The absolute value of (five minus two  $Z$ ) minus one equals eight.

### Answer One:

$Z$  equals negative two or  $Z$  equals seven.

### Steps to the answer:

First, isolate the absolute value on the left side of the equation by adding one to each side of the equation.

This results in the equation becoming the absolute value of (five minus two  $Z$ ) equals nine.

Next, set up to equations.

The first equation is five minus two  $Z$  equals nine and the second equation is five minus two  $Z$  equals negative nine.

Solve each equation for the variable  $Z$ .

In the first equation subtract five from both sides, which results in the equation becoming negative two  $Z$  equals four.

Divide each side of the equation by negative two to result in the first answer of  $Z$  equals negative two.

In the second equation, subtract five from both sides, which results in the equation becoming negative two  $Z$  equals negative fourteen.

Divide each side of the equation by negative two to result in the second answer of  $Z$  equals seven.

### Question Two:

The absolute value of ( $X$  plus five) minus seven equals negative two.

### Answer Two:

The answer is  $X$  equals zero or  $X$  equals negative ten.

### Question Three:

The absolute value of (five  $X$  minus one) equals negative two.

### Answer Three:

The answer is no solution. An absolute value cannot equal a negative number.

Note: an absolute value cannot equal a negative value. The absolute value of  $X$  equals negative two does not make any sense.

### Question Four:

The absolute value of ((one over two) times  $X$  minus three over four) equals one over four.

### Answer Four:

The answer is  $X$  equals two or  $X$  equals one.

### Question Five:

The absolute value of ( $Y$  minus one) equals the absolute value of (seven plus  $Y$ ).

### Answer Five:

The answer is  $Y$  equals negative three.

Note: Always check your answers!

### Steps to the answer:

First set up to equations. The first equation is  $Y$  minus one equals seven plus  $Y$ .

The second equation is  $Y$  minus one equals negative one times (seven plus  $Y$ ).

Solve each equation for  $Y$ . The first equation results in zero equals eight.

This is a false statement so there is no solution for the first equation.

Simplifying the second equation results in two  $Y$  equals negative six.

Then, divide each side by two to get the answer of  $Y$  equals negative three.

Hence,  $Y$  equals negative three is the only solution.

## Part E. Solving Exponential Equations.

Some properties you will need to be familiar with:

If  $A$  to the  $R$  power equals  $A$  to the  $S$  power, then  $R$  equals  $S$ .

If  $A$  to the  $R$  power equals  $B$  to the  $R$  power, then  $A$  equals  $B$ .

### Question One:

Ten to the  $X$  power equals one thousand.

### Answer One:

The answer is  $X$  equals three.

### Steps to the answer:

Rewrite one thousand as ten to the third power.

This results in the equation becoming ten to the  $X$  power equals ten to the third power.

Therefore  $X$  equals three.

### Question Two:

Ten to the (three  $X$  plus five) power equals one hundred.

### Answer Two:

The answer is  $X$  equals negative one.

### Question Three:

Two to the ( $X$  plus one) power equals one over eight.

### Answer Three:

The answer is  $X$  equals negative four.

### Question Four:

Three to the  $X$  squared power times (nine to the  $X$  power) equals one over three.

### Answer Four:

The answer is  $X$  equals negative one.

### Question Five:

Two to the letter  $X$  squared power times (four to the two  $X$  power) equals one over eight.

### Answer Five:

The answer is  $X$  equals negative one and negative three.

## Part F. Solving Logarithmic Equations.

Properties of logarithms to be familiar with:

- If  $\log$  base B of M equals  $\log$  base B of N, then M equals N.
- If  $\log$  base B of X equals Y, then this equation can be written in exponential form as B to the Y power equals X.
- $\log$  base B of (M times N) equals  $\log$  base B of M plus  $\log$  base B of N.
- $\log$  base B of (M over N) equals  $\log$  base B of M minus  $\log$  base B of N.
- $\log$  base B of M to the R power equals R times  $\log$  base B of M.
- Always check your answer!! Bases and arguments of logarithms cannot be negative.

### Question One:

$\log$  base two of (X plus five) equals  $\log$  base two of (one minus five X).

#### Answer One:

The answer is X equals negative two over three.

#### Steps to the answer:

Use the first property of logarithms to rewrite the equation as X plus five equals one minus five X.

Isolate the variable X which results in the equation 6 X equals negative four.

Divide each side of the equation by six to get X equals negative four over six which reduces to negative two over three.

### Question Two:

Two times  $\log$  base three of (X plus one) equals  $\log$  base three of (four X).

#### Answer Two:

The answer is X equals one.

#### Steps to the answer:

First, rewrite the left side of the equation as  $\log$  base three of (X plus one) to the second power.

Since both sides of the equation are in base three, rewrite the equation as (X plus one) to the second power equals four X.

Multiply the binomial on the left side of the equation and subtract four X on each side of the equation to get the quadratic equation of X to the second power minus two X plus one equals zero.

Solve this quadratic equation to get the answer of X equals one.

### Question Three:

$\log$  base two of (X plus one) plus  $\log$  base two of (X minus one) equals three.

#### Answer Three:

The answer is X equals three.

This is the only solution since negative three causes the argument of the logarithm to be negative.

### Question Four:

The natural log of X plus the natural log of (two X plus one) equals zero.

#### Answer Four:

The answer is X equals one over two.

This is the only solution since negative one causes the argument of a logarithm to be negative.

### Question Five:

The natural log of X plus the natural log of (X plus two) equals the natural log of three.

#### Answer Five:

The answer is X equals one.

This is the only solution since negative three causes the argument of a logarithm to be negative.

**Question Six:**

Three to the two X power equals four to the (X plus one) power.

**Answer Six:**

The answer is the fraction of X equals the natural log of four all over (two times the natural log of three minus the natural log of four)

**Steps to the answer:**

First, take the natural log of both sides of the equation.

Use the properties of logarithms to rewrite the left and right side of the equation.

This results in the equation becoming two X times the natural log of three equals (X plus one) times the natural log before.

Use the distributive property on the right side of the equation.

This results in equation becoming two X times the natural log of three equals X times the natural log before plus the natural log of four.

Next, subtract X times the natural log of four from each side of the equation.

This results in the equation of two X is a natural log of three minus X times the natural log of four equals the natural log of four.

Factor out an X from each term on the left side of the equation.

This results in the equation of X times (two times the natural log of three minus the natural log of four) equals the natural log four.

Divide each side of the equation by (two times the natural log of three minus natural log of four).

This results in the answer of X equals the natural log of four all over (two times the natural log of three minus the natural log four).

## Part G. Solving Radical Equations.

Steps to solving equations with radicals:

- One. Isolate the radical on one side of the equation and everything else on the other side.
- Two. If it is a square root, then square both sides. If it is a cube root, then cube both sides, etc....
- Three. Solve for the given variable and check your answer.

Note: a radical with an even index such as the square root, the fourth root, the sixth root, etc. Cannot have a negative argument (the square root can but you must use complex numbers).

### Question One:

Four times the square root of (two Y minus one) minus two equals zero.

### Answer One:

The answer is Y equals five over eight.

### Steps to the answer:

First, isolate the radical on one side of the equation by adding two to each side of the equation.

Then divide each side of the equation by four.

This results in the equation of the square root of (two Y minus one) equals one over two.

Square both sides of the equation to eliminate the square root.

This results in the equation of two Y minus one equals one over four.

Add one to each side of the equation to get two Y equals five over four.

Divide each side of the equation by two which results in the answer of Y equals five over eight.

### Question Two:

The square root of (two X plus one) plus five equals eight.

### Answer Two:

The answer is X equals four.

### Question Three:

The square root of (five X minus one) minus two times the square root of (X plus one) equals zero.

### Answer Three:

The answer is X equals five.

### Steps to the answer:

First, add two times the square root of (X plus one) to each side of the equation.

This results in the equation of the square root of (five X minus one) equals two times the square root of (X plus one).

Square both sides of the equation to eliminate the radical sign.

This results in the equation of five X minus one equals four times (X plus one).

Solve this equation for X to get the solution of X equals five.

### Question Four:

The square root of (X to the second power plus nine) plus X plus one equals zero.

### Answer Four:

The answer is no solution.

When working the problem, X equals four is a possible solution but it does not work in the original equation.

### Question Five:

The cube root of (three X plus two) plus four equals six.

### Answer Five:

The answer is X equals two.

### Question Six:

The fourth root of (W to the second power plus seven) equals two.

### Answer Six:

The answer is W equals three and negative three.

## Section Four: Solving Inequalities.

Solve the following inequalities and express the answer graphically and using interval notation.

### Part A. Solving Linear Inequalities.

When solving linear inequalities, use the same steps as solving an equation. The difference is when you multiply or divide both sides by a negative number, you must change the direction of the inequality. For example: consider the inequality of five is greater than three. Multiplying both sides of the inequality by negative one results in negative five is less than negative three.

#### Question One:

(Three over five) times X plus four is less than or equal to negative two.

#### Answer One:

The answer is X is less than or equal to negative ten.

The graph of the answer is a horizontal number line with a closed (or filled in) circle at negative ten with a shaded arrow pointing to the left to represent the less than or equal to.

The interval notation is: (negative infinity comma negative ten].

#### Steps to the answer:

First, subtract four from both sides of the inequality.

This results in the inequality of three over five X is less than or equal to negative six.

Then, multiply both sides of the equation by five over three, which results in the inequality of X is less than or equal to negative ten.

#### Question Two:

Three times (X plus three) is greater than or equal to five times (X minus one).

#### Answer Two:

The answer is X is less than or equal to seven.

The graph of the answer is a horizontal number line with a closed (or filled in) circle at seven with a shaded arrow pointing to the left to represent the less than or equal to.

The interval notation is: (negative infinity comma seven].

#### Question Three:

Three times (X plus two) minus six is greater than negative two times (X minus three) plus fourteen.

#### Answer Three:

The answer is X is greater than four.

The graph of the answer is a horizontal number line with an open (or not filled in) circle at four with a shaded arrow pointing to the right to represent the greater than.

The interval notation is: (four comma infinity).

#### Question Four:

Two is less than or equal to three X minus ten is less than or equal to five.

#### Answer Four:

The answer is for is less than or equal to X is less than or equal to five.

The graph of the answer is a horizontal number line with two closed (or filled in) circles at four and five.

The number line is shaded between four and five to represent the solutions.

The interval notation is: [four comma five].

## Part B. Solving Absolute Value Inequalities: Solve and Graph.

Think of the inequality sign as an alligator. If the alligator is facing away from the absolute value sign such as, the absolute value of  $X$  is less than five, that one can remove the absolute value and write the inequality of negative five is less than  $X$  is less than five. This expression indicates that  $X$  cannot be farther than five units away from zero.

If the alligator faces the absolute value such as, the absolute value of  $X$  is greater than five, then one can remove the absolute value in write the inequalities of  $X$  is greater than five or  $X$  is less than negative five. These expressions expressed that  $X$  cannot be less than five units away from zero.

### Question One:

The absolute value of (four  $X$  plus one) is less than or equal to six.

### Answer One:

The answer is negative seven over four is less than or equal to  $X$  is less than or equal to five over four.

The graph of the answer is a horizontal number line with two closed (or filled in) circles at negative seven over four and at five over four. The number line is shaded between negative seven over four and five over four to represent the solutions.

The interval notation is: [negative seven over four comma five over four]

### Steps to the answer:

Since the inequality sign is facing away from the absolute value sign, rewrite the inequality as negative six is less than or equal to four  $X$  plus one is less than or equal to six.

Isolate the variable  $X$  by subtracting one from each side of the inequality.

This results in the inequality of negative seven is less than or equal to four  $X$  is less than or equal to five.

Divide each side of the inequality by four.

This results in the answer of negative seven over four is less than or equal to  $X$  is less than or equal to five over four.

### Question Two:

The absolute value of (four  $X$  plus three) plus two is greater than nine.

### Answer Two:

The answer is  $X$  is greater than one or  $X$  is less than negative five over two.

The graph of the answer is a horizontal number line with two open (or not filled in) circles at negative five over two and at one.

At negative five over two there is a shaded arrow pointing to the left to represent the inequality of  $X$  is less than negative five over two.

At one, there is a shaded arrow pointing to the right to represent the inequality of  $X$  is greater than one.

The interval is: (negative infinity comma negative five over two) union (one comma infinity).

### Question Three:

The absolute value of ( $X$  plus five all over three) is greater than or equal to five.

### Answer Three:

The answer is  $X$  is less than or equal to negative twenty or  $X$  is greater than or equal to ten.

The graph of the answer is a horizontal number line with two closed (or filled in) circles at negative twenty and at ten.

At negative twenty there is a shaded arrow pointing to the left to represent the inequality of  $X$  is less than or equal to negative twenty.

At ten, there is a shaded arrow pointing to the right to represent the inequality of  $X$  is greater than or equal to ten.

The interval is: (negative infinity comma negative twenty] union [ten comma infinity).

### Question Four:

The absolute value of (five minus two  $X$ ) is less than fifteen.

### Answer Four:

The answer is negative five is less than  $X$  is less than 10.

The graph of the answer is a horizontal number line with two open (or not filled in) circles at negative five and ten.

The number line is shaded between negative five and ten to represent the solutions.

The interval is: (negative five comma ten).

## Part C. Solving Quadratic or Rational Inequalities.

Note: Go through question one to get detailed steps of solving quadratic and rational inequalities.

### Question One:

Three  $X$  to the second power minus eleven  $X$  minus four is less than zero.

### Answer One:

The answer is the interval (negative one over three comma four)

### Steps to the answer:

- Step one. Zero should be on one side of the inequalities while everything else is on the other side. This is already done for this problem.
- Step Two. Factor if possible. In this problem three  $X$  to the second power minus eleven  $X$  minus four factors into (three  $X$  plus one) times ( $X$  minus four).
- Step Three. Set the factors equal to zero and solve. In this problem we get  $X$  equals negative one over three and  $X$  equals four.
- Step four. Draw a chart using a horizontal. Then, use a vertical lines to divide regions on the number line that make the factors zero. Write the factors on the left side of the number line. The study guide gives in image of a horizontal number line split into thirds using two vertical lines. One vertical line represents the value of negative one over three and the second vertical line represents the value of four. Above the horizontal line and to the left are written in the factors three  $X$  plus one and  $X$  minus four.
- Step five in each region, pick a number in substituted in for  $X$  in each factor. Record the sign in that region. For example, in the region that represents all the numbers less than negative one over three we can pick a value such as negative one. If we substitute negative one into the factor of three  $X$  plus one, this results in a negative two. Therefore, in their region that represents less than negative one over three, we record a negative sign next to the factor of three  $X$  plus one. Completing this for each region creates a data picture of positive and negative signs. For the factor of three  $X$  plus one we have the following signs: the factor is negative for every value less than negative three, and the factor is positive for every value greater than negative one over three. For the factor of  $X$  minus four, it is negative for all numbers less than four and it is positive for all numbers greater than four.
- Step six. If  $X$  is a number in the first region (which is all numbers less than negative one over three) then both factors will be negative. Since a negative times a negative number is positive,  $X$  in the first region is not a solution. Continue with step five until you find a region that satisfies the inequality. For this problem that region is between negative one over three and four. This is because all numbers greater than negative one over three make the factor of three  $X$  plus one positive and all numbers less than four make the second factor of  $X$  minus four negative. A positive times a negative number is a negative number and all negative numbers are less than zero. Since the original inequality is wanting all numbers less than zero, we can use any number in the interval of (negative one over three, four)
- Step seven. Especially with rational expressions, check that your endpoints do not make the original inequality undefined.

### Question Two:

Six  $X$  of the second power plus five  $X$  is greater than or equal to four.

### Answer Two:

The answer is the interval: (negative infinity comma negative for over three] union [one over two comma infinity).

### Question Three:

( $X$  plus two) all over (three minus  $X$ ) is greater than or equal to zero.

### Answer Three:

The answer is the interval of: [negative two comma three).

### Question Four:

( $X$  plus one) times ( $X$  minus three) all over (two  $X$  plus seven) is less than or equal to zero.

### Answer Four:

The answer is the interval of: (negative infinity comma negative seven over two) union [negative one comma three].

## Section Five: Lines and Regions.

### Question One:

Find the X and Y intercepts, the slope, and the graph six X plus five Y equals thirty.

### Answer One:

The answer is: the X intercept is at (five, zero); the Y intercept is at (zero, six) and the slope is negative six over five.

The graph is on a coordinate plane with a continuous line that is decreasing from left to right that goes through the X and Y intercepts.

### Question Two:

Find the X and Y intercepts, the slope, and graph X equals three.

### Answer Two:

The answer is: the X intercept is at (three, zero); there is no Y intercept and there is no slope.

The graph is on a coordinate plane with a continuous vertical that goes through the X intercept.

### Question Three:

Find the X and Y intercepts, the slope, and graph Y equals negative four.

### Answer Three:

The answer is: there is no X intercept; the Y intercept is at (zero, negative four) and the slope is zero.

The graph is on a coordinate plane with a continuous horizontal line that goes through the Y intercept.

### Question Four:

Write in slope intercept form the line that passes through the points (four, six) and (negative four, two).

### Answer Four:

The answer is Y equals (one over two) times X plus four.

### Question Five:

Write in slope intercept form the line perpendicular to the graph of four X minus Y equals negative one and containing the point (two, three).

### Answer Five:

The answer is Y equals negative one over four X plus seven over two.

### Question Six:

Graph the solution set of X minus Y is greater than or equal to two.

### Answer Six:

The answer is the graph of a continuous line that is increasing from left to right that has an X intercept of (two, zero) and a Y intercept of (zero, negative two).

The coordinate plane is shaded under this line to represent all the solutions to the inequality of X minus Y is greater than or equal to two.

### Question Seven:

Graph the solution set of negative X plus three Y is less than negative six.

### Answer Seven:

The answer is the graph of a dotted line that is increasing from left to right that has an X intercept of (six, zero) and a Y intercept of (zero, negative two).

The coordinate plane is shaded under this line to represent all the solutions to the inequality of negative X plus three Y is less than negative six.

## Section Six: Graphing Relations, Domain and Range.

For each relation, state if it is a function, state the domain and range, and graph it.

### Question One:

Y equals the square root of (X plus two).

### Answer One:

The answer is; yes, this is a function; the domain is  $[-2, \infty)$ ; the range is  $[0, \infty)$

The graph is of a curve that increases from left to right that starts at the point  $(-2, 0)$ .

The curve has a Y intercept of  $(0, \sqrt{2})$ .

Other coordinate points on the curve are  $(2, 2)$  and  $(7, 3)$ .

### Question Two:

Y equals the square root of (X) minus two

### Answer Two:

The answer is; yes, this is a function; the domain is  $[0, \infty)$ ; the range is  $[-2, \infty)$

The graph is of a curve that increases from left to right that starts at the point  $(0, -2)$ .

The curve has a X intercept of  $(4, 0)$ . Another coordinate point on the curve is  $(9, 1)$ .

### Question Three:

Y equals (X minus one) all over (X plus two).

### Answer Three:

The answer is; yes, this is a function; the domain is all real numbers except for negative two; the range is  $(-\infty, 1) \cup (1, \infty)$ .

The graph has two curves. There is a vertical asymptote on the line X equals negative two and a horizontal asymptote at Y equals one.

One curve is increasing from left to right and is located to the left of the vertical asymptote and above the horizontal asymptote.

The first curve does not have an X or Y intercept.

The second curve is increasing from left to right and is located to the right of the vertical asymptote and below the horizontal asymptote.

The second curve has a Y intercept of  $(0, -\frac{1}{2})$  and a X intercept of  $(1, 0)$ .

### Question Four:

F of X equals negative absolute value of (X plus one) plus three

### Answer Four:

The answer is; yes, this is a function; the domain is  $(-\infty, \infty)$ ; the range is  $(-\infty, 3]$ .

The graph is a V shaped graph opening downward.

The vertex of the graph is at the coordinate point  $(-1, 3)$ .

The graph has a Y intercept of  $(0, 2)$  and it has two X intercepts at  $(-4, 0)$  and  $(2, 0)$ .

### Question Five:

F of X equals (two X minus five) all over (X to the second power minus nine).

### Answer Five:

The answer is; yes, this is a function; the domain is all real numbers except for positive and negative three; the range is all real numbers.

The graph contains three curves.

There are two vertical asymptotes one on the line X equals negative three and another at X equals three.

There is a horizontal asymptote at Y equals zero.

Reading the graph from left to right, the first curve is decreasing from left to right and is located to the left of the vertical asymptote of X equals negative three and below the horizontal asymptote. The first curve does not have an X or Y intercept.

The second curve is an S shaped curve decreasing from left to right and is located in between the two vertical asymptote and crosses through the horizontal asymptote.

The second curve has a Y intercept of  $(0, \frac{5}{9})$  and has an X intercept of  $(\frac{5}{2}, 0)$ .

The third curve is decreasing from left to right and is located to the right of the vertical asymptote of X equals three and is above the horizontal asymptote. The third curve does not have an X or Y intercepts.

### Question Six:

X equals Y to the second power plus two.

### Answer Six:

The answer is; no, this is not a function; the domain is  $[2, \infty)$ ; the range is  $(-\infty, \infty)$ .

The graph is a parabola (U shaped graph) opening to the right.

The vertex and x intercept of the graph is at the coordinate point  $(2, 0)$ .

The graph has a no Y intercept.

Two other coordinate points on the graph are at  $(3, 1)$  and  $(3, -1)$ .

**Question Seven:**

Y equals X to the second power plus eight X minus six.

**Answer Seven:**

The answer is; yes, this is a function; the domain is (negative infinity, infinity); the range is [negative twenty two, infinity).

The graph is a parabola (U shaped graph) opening upward.

The vertex is at the coordinate point (negative four, negative twenty two).

The graph has a Y intercept of (zero, negative six).

**Question Eight:**

Y equals the square root of (negative X).

**Answer Eight:**

The answer is; yes, this is a function; the domain is (negative infinity, zero]; the range is [zero, infinity)

The graph is of a curve that decreases from left to right and ends at the coordinate point (zero, zero).

Other coordinate points on the curve are (negative four, two) and (negative nine, three).

**Question Nine:**

Y equals three to the X power.

**Answer Nine:**

The answer is; yes, this is a function; the domain is (negative infinity, infinity); the range is (zero, infinity)

The graph is of an exponential curve that increases from left to right.

The Y intercept is at (zero, one).

Other coordinate points on the curve are (one, three), (two, nine), (negative one, one over three) and (negative two, one over nine).

**Question Ten:**

H of X equals six X of the second power all over (three X to the second power minus two X minus one).

**Answer Ten:**

The answer is; yes, this is a function; the domain is all real numbers except for negative one over three and one; the range is (negative infinity, zero] union (two, infinity).

The graph contains three curves.

There are two vertical asymptotes one on the line X equals negative one over three and another at X equals one.

There is a horizontal asymptote at Y equals two.

Reading the graph from left to right, the first curve is increasing from left to right and is located to the left of the vertical asymptote of X equals negative one over three and above the horizontal asymptote.

The first curve does not have an X or Y intercept.

The second curve is a U shaped curve opening downward and is located in between the two vertical asymptote and below the horizontal asymptote.

The second curve has an X and Y intercept of (zero, zero).

The third curve is decreasing from left to right and is located to the right of the vertical asymptote of X equals one and is above the horizontal asymptote.

The third curve does not have an X or Y intercept.

## Section seven: Exponents and Radicals.

Simplify. Assume all variables are greater than zero. Rationalize the denominators when needed.

### Question One:

The cube root of (negative eight  $X$  to the third power).

### Answer One:

The answer is negative two  $X$ .

### Question Two:

Five times the square root of one hundred forty seven minus four times the square root of forty eight.

### Answer Two:

The answer is nineteen times the square root of three.

### Steps to the answer:

First, simplify five times the square root of one hundred forty seven as thirty five times the square root of three.

Also simplify four times the square root of forty eight as sixteen times the square root of three.

This results in the expression of thirty five times the square root of three minus sixteen times the square root of three which equals nineteen times the square root of three.

### Question Three:

The square root of five times (the square root of fifteen minus the square root of three).

### Answer Three:

The answer is five times the square root of three minus the square root of fifteen.

### Question Four:

( $X$  to the two over three power  $Y$  to the negative four over three power all over  $X$  to the negative five over three power) to the third power.

### Answer Four:

The answer is the fraction  $X$  to the seventh power over  $Y$  to the fourth power.

### Question Five:

The cube root of (forty  $X$  of the fourth all over  $Y$  to the ninth).

### Answer Five:

The answer is the fraction two  $X$  times the square root of (five  $X$ ) all over  $Y$  to the third power.

### Question Six:

(fifty four  $A$  to the negative six power  $B$  to the second power all over nine  $A$  to the negative three power  $B$  to the eighth power) to the negative two power.

### Answer Six:

The answer is the fraction of  $A$  to the six power  $B$  to the twelfth power all over thirty six.

### Steps to the answer:

First, simplify the expression inside parentheses before applying the negative two power.

This results in the expression of [six over ( $A$  to the third power  $B$  to the sixth power)] all raised to the negative two power.

Remember, that negative exponents represent a reciprocal of the fraction.

Then, raised each variable and number to the second power.

This results in the answer of  $A$  to the six power  $B$  to the twelfth power all over thirty six.

### Question Seven:

The cube root of (twenty seven  $A$  to the third power) all over the cube root of (two  $A$  to the second power  $B$  to the second power).

### Answer Seven:

The answer is the fraction three times the cube root of (four  $A B$ ) all over two  $B$ .

### Steps to the answer:

First, simplify the numerator of the problem.

This results in the expression of three  $A$  all over the cube root of (two  $A$  to the second power  $B$  to the second power).

Multiply the numerator and denominator by the fraction of the cube root of (four  $A B$ ) all over the cube root of (four  $A B$ ).

Doing this will rationalize the denominator and results in the expression of three  $A$  times the cube root of (four  $A B$ ) all over (two  $A B$ ).

Simplify the fraction to get the answer of three times the cube root of (four  $A B$ ) all over two  $B$ .

### Question Eight:

Two over (the square root of five minus the square root of three).

### Answer Eight:

The answer is the square root of five plus the square root of three.

**Question Nine:**

X over (the square root of (X) plus three).

**Answer Nine:**

The answer is the fraction of (X times the square root of X minus three X) all over (X minus nine).

**Steps to the answer:**

Rationalize the denominator by multiplying the fraction by another fraction of (the square root of X minus three) all over (the square root of X minus three).

After simplifying by using the distributive property and combining like terms, we get the answer.

## Section Eight: Complex Numbers.

Perform the indicated operation and simplify.

### Question One:

The square root of negative sixteen minus four times the square root of negative nine.

### Answer One:

The answer is negative eight I.

### Steps to the answer:

Rewrite the square root of negative sixteen as four I and the square root of negative nine as three I.

This results in the expression of four I minus four times three I.

Simplify the expression further to get four I minus twelve I which equals negative eight.

### Question Two:

The square root of negative sixteen times the square root of negative nine.

### Answer Two:

The answer is negative twelve.

### Steps to the answer:

Rewrite the square root of negative sixteen as four I and the square root of negative nine as three I.

Then, multiply four I times three I which equals twelve I to the second power.

Remember, that I to the second power equals negative one. Therefore twelve times negative one equals negative twelve.

### Question Three:

The square root of negative sixteen over the square root of negative nine.

### Answer Three:

The answer is four over three.

### Steps to the answer:

Rewrite the square root of negative sixteen as four I in the square root of negative nine as three I.

This results in the expression of four I over three I.

Next, rationalize the denominator by multiplying four I over three I by the fraction three I over three I.

This results in the fraction of twelve I to the second power over nine I to the second power.

Using the fact that I to the second power equals negative one results in the expression of negative twelve over negative nine which simplifies to the answer of four over three.

### Question Four:

(four minus three i) times (four plus three i).

### Answer Four:

The answer is twenty five.

### Steps to the answer:

Use the distributive property twice to simplify the two binomials to get the expression of sixteen minus nine I to the second power.

Using the fact that I to the second power equals negative one results in the expression of sixteen plus nine which equals the answer of twenty five.

### Question Five:

(four minus three i) to the second power.

### Answer Five:

The answer is seven minus twenty four I.

### Steps to the answer:

Rewrite the problem as the two binomials (four minus three I) times (four minus three I).

Use the distributive property twice to multiply the two binomials to get sixteen minus twenty four I plus nine I to the second power.

Use the fact that I to the second power equals negative one to simplify the expression into sixteen minus twenty four I minus nine.

Combining like terms results in the answer of seven minus twenty four I.

**Question Six:**

$i$  to the twenty fifth power.

**Answer Six:**

The answer is  $I$ .

**Steps to the answer:**

Rewrite  $I$  to the twenty fifth power as  $I$  times  $I$  to the twenty fourth power.

Rewrite  $I$  to the twenty fourth power as  $(I$  to the second power) to the twelfth power.

This results in the expression of  $a$  times  $(I$  to the second power) to the twelfth power.

Rewrite  $I$  to the second power as negative one. This results in the expression of  $I$  times (negative one) to the twelfth power.

Negative one to the twelfth power equals positive one and  $I$  times one equals  $I$ .

**Question Seven:**

$(3 - 2i)$  all over  $(4 + 5i)$ .

**Answer Seven:**

The answer is  $\frac{2 - 23i}{41}$ .

**Steps to the answer:**

To rationalize the denominator multiply the fraction  $(3 - 2i)$  all over  $(4 + 5i)$  by  $(4 - 5i)$  all over  $(4 - 5i)$ .

Simplifying the numerator and denominator results in the fraction of  $(12 - 23I + 10I^2)$  all over  $(16 - 25I^2)$ .

Using the fact that  $I$  to the second power equals negative one results in the fraction of  $(12 - 23I - 10)$  all over  $(16 + 25)$ .

Combining like terms results in the answer of  $\frac{2 - 23I}{41}$ .

## Section Nine: Exponential Functions and Logarithms.

### Question One:

Graph F of X equals three to the X power plus one.

### Answer One:

The graph is of an exponential curve that increases from left to right. The Y intercept is at (zero, two).

Other coordinate points on the curve are (one, four), (two, ten), (negative one, four over three) and (negative two, ten over nine).

### Question Two:

Graph G of X equals two to the (X minus one) power.

### Answer Two:

The graph is of an exponential curve that increases from left to right. The Y intercept is at (zero, one over two).

Other coordinate points on the curve are (one, one), (two, two), (negative one, one over four) and (negative two, one over eight).

### Question Three:

Express eight to the negative to power equals one over sixty four in logarithmic form.

### Answer Three:

The answer is log base eight of one over sixty four equals negative two.

### Question Four:

Express log base five of twenty five equals two and exponential form.

### Answer Four:

The answer is five to the second power equals twenty five.

### Question Five:

Solve log base two of X equals four.

### Answer Five:

The answer is X equals sixteen.

### Steps to the answer:

First, rewrite the equation in exponential form.

This results in the equation two to the fourth power equals X.

Simplify two to the fourth power to get sixteen, so X equals sixteen.

### Question Six:

Solve log base X of nine equals two.

### Answer Six:

The answer is X equals three; negative three is not a solution because bases are not allowed to be negative.

### Question Seven:

Graph H of X equals log base three of X.

### Answer Seven:

The graph is of a logarithmic curve that increases from left to right.

There is no Y intercept. The X intercept is at (one, zero).

Other coordinate points on the curve are (three, one), (nine, two), (one over three, negative one) and (one over nine, negative two).

### Question Eight:

Use the properties of logarithms to expand as much as possible: log base four of (three over Y).

### Answer Eight:

The answer is log base four of three minus log base four of Y

**Question Nine:**

How long will it take eight hundred fifty dollars to be worth one thousand dollars if it is invested at twelve percent interest compounded quarterly?

**Answer Nine:**

The answer is the fraction  $T$  equals  $\log$  of (twenty over seventeen) all over four times  $\log$  of (one plus (twelve hundredths over four)).

Note: An approximate answer can be found by using a calculator.

Steps to the answer: Use the compound interest formula of  $A$  equals  $P$  times (one plus ( $R$  over  $N$ )) to the  $NT$  power where:

$A$  equals the money ended with

$P$  equals the money started with

$R$  equals the yearly interest rate

$N$  equals the number of compounds per year

$T$  equals the number of years

Substitute the values given in the question into the equation.

This results in one thousand equals eight hundred fifty times (one plus (twelve hundredths over four)) to the four times  $T$  power.

To find the time, solve the equation for  $T$ .

First, divide each side of the equation by eight hundred fifty.

This results in the equation twenty over seventeen equals (one plus (twelve hundredths over four)) to the four times  $T$  power.

Next, take the  $\log$  of base ten on both sides of the equation.

This results in the equation  $\log$  of (twenty over seventeen) equals  $\log$  of (one plus (twelve hundredths over four)) to the four times  $T$  power.

Use properties of logarithms to rewrite the equation as  $\log$  of (twenty over seventeen) equals four  $T$  times  $\log$  of (one plus (twelve hundredths over four)).

Divide both sides of the equation by four times  $\log$  of (one plus (twelve hundredths over four)) to get the variable  $T$  isolated.

This results in the answer of  $T$  equals  $\log$  of (twenty over seventeen) all over four times  $\log$  of (one plus (twelve hundredths over four)).

## Section Ten: Systems of Equations and Matrices.

**Question One:** Solve the system:

Two X plus three Y equals seven and

Six X minus Y equals one.

**Answer One:**

The answer is X equals one over two and Y equals two. This can also be expressed as the coordinate point (one over two, two).

**Question Two:** Solve the system:

X plus two Y plus two Z equals three

Two X plus three Y plus six Z equals two and

Negative X plus Y plus Z equal zero.

**Answer Two:**

(one, two, negative one)

**Question Three:**

Perform the indicated matrix operation:

Negative two times the two by two matrix where the first row has entries three and one and the second row has entries of negative one and two plus three times the two by two matrix where the first row has entries one over three and negative two and the second row has entries one and six.

**Answer Three:**

The answer is a two by two matrix where the first row has entries of negative five and negative eight.

The second row has entries of five and fourteen.

**Question Four:**

Multiply the matrices:

The first matrix is a three by three matrix where the first row has entries of one, negative one, and one.

The second row has entries of zero, two and zero. The third row has entries of negative two, one and negative three.

The second matrix is a three by three matrix where the first row has entries of zero, two and one.

The second row has entries of one, two and zero. The third row has entries of zero, zero and one.

**Answer Four:**

The answer is the three by three matrix where the first row has entries of negative one, zero and two.

The second row has entries of two, four and zero. The third row has entries of one, negative two, negative five.

**Question Five:**

Find the determinant of the two by two matrix where the first row has entries of one and negative two.

The second row has entries of three and negative one.

**Answer Five:**

The answer is five.

**Question Six:**

Find the inverse of the two by two matrix where the first row has entries of one and two.

The second row has entries of negative one and two.

**Answer Six:**

The answer is the two by two matrix where the first row has entries of one over two and negative one over two.

The second row has entries of one over four and one over four.

## Section Eleven: Story Problems.

### Question One:

Sam made a ten dollars more than twice what Pete earned in one month.

If together they earned seven hundred sixty dollars, how much did each earn that month?

### Answer One:

The answer is that Pete earns two hundred fifty dollars and Sam earns five hundred ten dollars.

Steps to the answer: Let  $X$  equal the money Pete Burns and let two  $X$  plus ten equal the money Sam.

The equation representing their combined money is (two  $X$  plus ten) plus  $X$  equals seven hundred sixty.

Solve the equation for  $X$  to find how much he earns and then use that amount to find how much Sam earns.

### Question Two:

A woman burns up three times as many calories running as she does when walking the same distance.

If she runs two miles and walks five miles turn burn up a total of seven hundred seventy calories, how many calories will she burn up while running one mile?

### Answer Two:

The answer is two hundred ten calories.

Steps to the answer:

Let  $X$  equal burn calories while walking and let three  $X$  equal burn calories while running.

The equation representing total calories burned is two times (three  $X$ ) plus five  $X$  equals seven hundred seventy.

Solving for  $X$  gives the solution of  $X$  equals seventy. This represents the calories burned while walking.

Multiply seventy by three to get the number of calories burned while running.

### Question Three:

A pole is standing in a small lake.

If one sixth of the length of the pole is in the sand at the bottom of the lake, twenty five feet are in the water, and two thirds of the total length is in the air above the water, what is the length of the pole?

The question also has a diagram where a rectangle represents the pole.

A horizontal line is drawn about one fourth from the bottom of the rectangle to represent how much of the pole is in the sand.

A second horizontal line is drawn about one fourth from that top of the rectangle to represent how much of the pole is in the water.

### Answer Three:

The answer is one hundred fifty feet.

Steps to the answer: Let  $X$  equal the length of the pole.

The equation two over three  $X$  plus twenty five plus one over six  $X$  equals  $X$  represents the length of the pole separated into the parts under the sand and under the water.

Solve the equation for  $X$  to get the answer.

## Section Twelve: Conic Sections.

Part One: Graph the following, and find the center, foci, and asymptotes if possible.

### Question One:

$(X - 2)^2 + Y^2 = 16$ .

### Answer One:

The center is at the coordinate point  $(2, 0)$ ; the radius has a length of four.

There are no foci or asymptotes.

The graph is a circle on the coordinate plane with a center at  $(2, 0)$ .

The radius length of four is used to draw the outer edge of the circle.

Four coordinate points on the circle are  $(6, 0)$ ,  $(-2, 0)$ ,  $(2, 4)$  and  $(2, -4)$ .

### Question Two:

$\frac{(X + 1)^2}{16} + \frac{(Y - 2)^2}{9} = 1$ .

### Answer Two:

This is an equation of an ellipse.

The center is at the coordinate point  $(-1, 2)$  and has foci at the points  $(-1 + \sqrt{7}, 2)$  and  $(-1 - \sqrt{7}, 2)$ . There are no asymptotes.

The graph is an ellipse on the coordinate plane with a center at  $(-1, 2)$ .

From the center, move left four units to plot a point at  $(-5, 2)$  and right four units to plot a point at  $(3, 2)$ .

Then, move up three units from the center to plot a point at  $(-1, 5)$  and down three units from the center to plot a point at  $(-1, -1)$ . Connect these four points to sketch a graph of the ellipse.

### Question Three:

$\frac{(X + 1)^2}{16} - \frac{(Y - 2)^2}{9} = 1$ .

### Answer Three:

This is an equation of hyperbola.

The center is at the coordinate point  $(-1, 2)$  and has foci at the points  $(-6, 2)$  and  $(4, 2)$ .

There are two asymptotes. One is the equation  $Y = \frac{3}{4}X - \frac{3}{4}$ .

The other asymptote is the equation  $Y = -\frac{3}{4}X + \frac{5}{4}$ .

The graph is a hyperbola on the coordinate plane with a center at  $(-1, 2)$ .

Graph the two asymptotes, which both intersect at the center and forms an X shape.

The hyperbola opens to the left and the right within the X shape formed by the asymptotes.

Note: The graph can also be described as two parabolas. One opens to the left and the other one opens to the right.

### Question Four:

$(X - 2)^2 + Y = 4$ .

### Answer Four:

This is an equation of a parabola. The vertex is at the coordinate point  $(2, 4)$ .

The focus is at the coordinate point  $(2, \frac{17}{4})$ . The directrix is the equation  $Y = \frac{17}{4}$ .

The graph is a parabola (U shaped graph) on the coordinate plane that opens downward.

The vertex is at the coordinate point  $(2, 4)$ . The graph has a Y intercept at  $(0, 0)$ .

The graph has two X intercepts at  $(0, 0)$  and  $(4, 0)$ .

Part Two: identify the conic section and put it into standard form.

### Question One:

$X^2 - 4X - 12 + Y^2 = 0$ .

### Answer One:

The answer is a circle where the equation in standard form is  $(X - 2)^2 + Y^2 = 16$ .

### Question Two:

$9X^2 + 18X + 16Y^2 - 64Y = 71$ .

### Answer Two:

The answer is an ellipse where the equation in standard form is  $[\frac{(X + 1)^2}{16}] + [\frac{(Y - 4)^2}{9}] = 1$ .

**Question Three:**

Nine X of the second power plus eighteen X minus sixteen Y to the second power plus sixty four Y equals one hundred ninety nine.

**Answer Three:**

The answer is a hyperbola where the equation in standard form is  $\frac{(X + 1)^2}{16} - \frac{(Y - 2)^2}{9} = 1$ .

**Question Four:**

X to the second power plus Y minus four X equals zero.

**Answer Four:**

The answer is a parabola where the equation in standard form is  $Y = -1(X - 2)^2 + 4$ .

## Section Thirteen: Sequences and Series.

### Question One:

Write out the first four terms of the sequence whose general term is  $A_{\text{subscript } N} = 3N - 2$ .

### Answer One:

The answer are the terms one, four, seven, ten.

### Question Two:

Write out the first four terms of the sequence whose general term is  $A_{\text{subscript } N} = N^2 - 1$ .

### Answer Two:

The answer are the terms zero, three, eight, fifteen.

### Question Three:

Write out the first four terms of the sequence whose general term is  $A_{\text{subscript } N} = 2N + 1$ .

### Answer Three:

The answer are the terms three, five, nine, seventeen.

### Question Four:

Find the general term for the following sequence: two, five, eight, eleven, fourteen, seventeen,...

### Answer Four:

The answer is  $A_{\text{subscript } N} = 3N - 1$

### Question Five:

Find the general term for the following sequence: four, two, one, one over two, one over four,...

### Answer Five:

The answer is  $A_{\text{subscript } N} = 4 \times (1/2)^{N-1}$  or  $(2)^{3-N}$ . This can also be written as  $(2)^{3-N}$ .

**Note:** Questions six and seven use summation notation, which uses the greek letter sigma.

### Question Six:

Find the sum:  $\sum_{K=0}^6 (2K - 1)$

### Answer Six:

The answer is thirty five.

### Steps to the answer:

First, find the sequence of numbers by substituting integer values for  $K$  starting at zero to four.

For example, substitute zero into the expression  $2K - 1$  for  $K$ . This results in negative one.

When substituting one into the expression  $2K - 1$  for  $K$ , this results in one.

Continue this for  $K$  values of two, three, four, five and six.

This results in the sequence of negative one, one, three, five, seven, nine and eleven. Add these numbers to get the sum of thirty five.

### Question Seven:

Expand the following:  $\sum_{K=0}^4 (4 \times X^K) \times Y^{4-K}$

### Answer Seven:

The answer is  $Y^4 + 4XY^3 + 6X^2Y^2 + 4X^3Y + X^4$

## Section Fourteen: Functions.

Let  $F$  of  $X$  equal two  $X$  plus nine and  $G$  of  $X$  equal sixteen minus  $X$  to the second power. Find the following.

### Question One:

$F$  of negative three plus  $G$  of two.

### Answer One:

The answer is fifteen.

### Steps to the answer:

First, find  $F$  of negative three, which equals three. Then, find  $G$  of two, which equals twelve. Add three and twelve to get the answer of fifteen.

### Question Two:

$F$  of five minus  $G$  of four.

### Answer Two:

The answer is nineteen

### Steps to the answer:

First, find  $F$  of five, which equals nineteen. Then, find  $G$  of four, which equals zero. Subtract nineteen and zero to get the answer of nineteen.

### Question Three:

$F$  of negative one times  $G$  of negative two.

### Answer Three:

The answer is eighty four.

### Steps to the answer:

First, find  $F$  of negative one, which equals seven. Then, find  $G$  of negative two, which equals twelve. Multiply seven and twelve to get the answer of eighty four.

### Question Four:

$F$  of five divided by  $G$  of five.

### Answer Four:

The answer is negative nineteen over nine.

### Steps to the answer:

First, find  $F$  of five, which equals nineteen. Then, find  $G$  of five, which equals negative nine. Divide nineteen and negative nine to get the answer of negative nineteen over nine.

### Question Five:

( $G$  composed with  $F$ ) of negative two.

### Answer Five:

The answer is negative nine.

### Steps to the answer:

A composition problem can be rewritten as  $G$  of  $F$  of negative two.

First, find  $F$  of negative two, which equals five.

Then, find  $G$  of five, which equals negative nine.

### Question Six:

$F$  of  $G$  of  $X$ .

**Note:** This is also written as  $F(G(X))$

### Answer Six:

The answer is negative two  $X$  to the second power plus forty one.

Steps to the answer:

First, replace  $G$  of  $X$  with its corresponding function.

This results in the expression  $F$  of (sixteen minus  $X$  to the second power).

Then, substitute (sixteen minus  $X$  to the second power) into the function  $F$  of  $X$ .

This results in the function negative two times (sixteen minus  $X$  to the second power) plus nine.

Simplify to get the answer of negative two  $X$  to the second power plus forty one.

**Question Seven:**

F inverse of two.

**Note:** This is also written as F to the negative one power (two).

**Answer Seven:**

The answer is negative seven over two.

**Steps to the answer:**

First, find the inverse of the function F of X, which is F inverse of X equals X minus nine all over two.

Then, substitute two into the inverse function.

This results in two minus nine all over two, which equals the answer of negative seven over two.

**Question Eight:**

F of F inverse of three.

**Note:** This is also written as F (F to the negative one power (three)).

**Answer Eight:**

The answer is three.

## Section Fifteen: Fundamental Counting Rule, Factorials, Permutations, and Combinations.

### **Question One:**

Evaluate: Eight factorial over (three factorial times (eight minus three) factorial).

### **Answer One:**

The answer is fifty six.

### **Question Two:**

A particular new car model is available with five choices of color, three choices of transmission, four types of interior, and two types of engines. How many different variations of this model car are possible?

### **Answer Two:**

The answer is one hundred twenty.

### **Question Three:**

In a horse race, how many different finishes among the first three places are possible for a ten horse race?

### **Answer Three:**

The answer is seven hundred twenty.

### **Question Four:**

How many ways can a three person subcommittee be selected from a committee of seven people? How many ways can a president, vice president, and secretary be chosen from a committee of seven people?

### **Answer Four:**

The answer is thirty five ways for the committee and two hundred ten ways for the election of president, vice president, and secretary.

## Section Sixteen: Trigonometry.

### Question One:

Graph the following through one period:  $F$  of  $X$  equals sine of  $X$ .

### Answer One:

The graph of the sine function is a continuous smooth curve with various increases and decreases.

One period is described as the  $X$  values from zero to two  $\pi$ .

The sine graph starts at the coordinate point (zero, zero). The graph increases to the coordinate point ( $\frac{\pi}{2}$ , one).

Then, the graph decreases to the coordinate point ( $\frac{3\pi}{2}$ , negative one).

The graph crosses the  $X$  axis at the coordinate point ( $\pi$ , zero). Finally, the graph increases to the coordinate point ( $2\pi$ , zero).

### Question Two:

Graph the following through one period:  $G$  of  $X$  equals cosine of ( $2X$ ).

### Answer Two:

The graph of this cosine function is a continuous smooth curve with various increases and decreases.

One period is described as the  $X$  values from zero to  $\pi$ .

This cosine graph starts at the coordinate point (zero, one).

The graph decreases to the coordinate point ( $\frac{\pi}{2}$ , negative one).

Then, the graph increases to the coordinate point ( $\pi$ , one).

The graph crosses the  $X$  axis at the coordinate points ( $\frac{\pi}{4}$ , zero) and ( $\frac{3\pi}{4}$ , zero).

### Question Three:

A man whose eye level is six feet above the ground stands forty feet from a building. The angle of elevation from high level to the top of the building is seventy two degrees. How tall is the building?

This question includes a diagram of a rectangular building.

To the left of the building is a man labeled as six feet tall.

A horizontal line is drawn from the man's head to the building and is labeled forty feet long.

A diagonal line is drawn from the man to the top of the building.

The angle between the horizontal line of forty feet and the diagonal line is labeled as seventy two degrees

### Answer Three:

The answer is approximately one hundred twenty nine point one feet.

Steps to the answer:

use the equation of  $X$  equals the height of the man plus the distance from the building times the tangent of the angle of elevation.

Substituting the values from the problem gives any equation of  $X$  equals six plus forty times the tangent of seventy two which approximately equals one hundred twenty nine point one.

### Question Four:

A man standing at the top of the sixty five meter lighthouse observes two boats.

Using the data given in the picture, determine the distance between the two boats.

The picture gives a lighthouse that is sixty five meters in height.

A horizontal line is drawn from the top of the lighthouse to represent the man looking horizontally out of the lighthouse.

Two diagonal lines are drawn from the top of the lighthouse to two boats left of the lighthouse.

The two boats are next to each other, but there is no information about how far the boats are from the lighthouse.

One diagonal line is forty eight degrees from the horizontal line.

This represents the man looking down forty eight degrees to see the first boat.

The second diagonal line is labeled as being six degrees ten minutes from the first diagonal line.

This represents the angle the man must look downward from the first boat to see the second boat.

Use the description of the picture to find the distance between the two boats.

### Answer Four:

The answer is the boats are eleven point five nine meters apart.

### Steps to the answer:

The distance between the boats is equal to sixty five times the tangent of forty two minus sixty five times the tangent of thirty five degrees and fifty minutes.

Using a calculator results in the answer of eleven point five nine.